Regular Behaviours with Names
On the Rational Fixpoint of Endofunctors on Nominal Sets

Stefan Milius, Lutz Schröder, Thorsten Wißmann

December 1, 2015

Last update: December 1, 2015
Regular Behaviours with Names
On the Rational Fixpoint of Endofunctors on Nominal Sets
Regular Behaviours with Names

On the Rational Fixpoint of Endofunctors on Nominal Sets
Regular Behaviours with Names

States & Transitions
(Functor-) Coalgebras

On the Rational Fixpoint of Endofunctors on Nominal Sets
Regular Behaviours with Names

States & Transitions
(Functor-) Coalgebras

Fresh Names, Renaming & Binding

On the Rational Fixpoint of Endofunctors on Nominal Sets
Regular Behaviours with Names

- Finite Description
- Finitely Presentable Objects (orbit-finite in Nom)
- States & Transitions
- (Functor-) Coalgebras
- Fresh Names, Renaming & Binding

On the Rational Fixpoint of Endofunctors on Nominal Sets
Regular Behaviours with Names

Finite Description

Finitely Presentable Objects (orbit-finite in Nom)

States & Transitions (Functor-) Coalgebras

Fresh Names, Renaming & Binding

On the Rational Fixpoint of Endofunctors on Nominal Sets
The Framework of Nominal Sets

Finite permutations on \( \mathcal{V} \)

Support for a \( \mathcal{S}_f(\mathcal{V}) \)-action \( \cdot : \mathcal{S}_f(\mathcal{V}) \times X \rightarrow X \)

"\( S \subseteq \mathcal{V} \) supports \( x \in X \)", if for all \( \pi \in \mathcal{S}_f(\mathcal{V}) \)

\[
\pi \text{ fixes } S \quad \implies \quad \pi \text{ fixes } x
\]

\( \pi(v) = v \quad \forall v \in S \)

\( \pi \cdot x = x \)
The Framework of Nominal Sets

Finite permutations on $\mathcal{V}$

Support for a $\mathcal{S}_f(\mathcal{V})$-action $\cdot : \mathcal{S}_f(\mathcal{V}) \times X \to X$

“$S \subseteq \mathcal{V}$ supports $x \in X$”, if for all $\pi \in \mathcal{S}_f(\mathcal{V})$

\[
\pi \text{ fixes } S \quad \Rightarrow \quad \pi \text{ fixes } x
\]

\[
\pi(v) = v \quad \forall v \in S \quad \pi \cdot x = x
\]

$(X, \cdot)$ a Nominal Set

“$\cdot$” a $\mathcal{S}_f(\mathcal{V})$-action & every $x \in X$ finitely supported

$(X, \cdot)$ a Nominal Set
The Framework of Nominal Sets

Finite permutations on \( \mathcal{V} \)

Support for a \( \mathcal{S}_f(\mathcal{V})\)-action \( \cdot : \mathcal{S}_f(\mathcal{V}) \times X \to X \)

"\( S \subseteq \mathcal{V} \) supports \( x \in X \)", if for all \( \pi \in \mathcal{S}_f(\mathcal{V}) \)

\[
\begin{align*}
\pi \text{ fixes } S & \implies \pi \text{ fixes } x \\
\pi(v) &= v \quad \forall v \in S & \pi \cdot x &= x
\end{align*}
\]

\((X, \cdot)\) a Nominal Set

"\( \cdot \)" a \( \mathcal{S}_f(\mathcal{V})\)-action & every \( x \in X \) finitely supported

\( x, y \) in the same orbit of \((X, \cdot)\)

if there is \( \sigma \) with \( \sigma \cdot x = y \).
The Framework of Nominal Sets

Finite permutations on $\mathcal{V}$

Support for a $\mathcal{G}_f(\mathcal{V})$-action $\cdot : \mathcal{G}_f(\mathcal{V}) \times X \to X$

"$S \subseteq \mathcal{V}$ supports $x \in X$", if for all $\pi \in \mathcal{G}_f(\mathcal{V})$

$\pi$ fixes $S$ $\implies$ $\pi$ fixes $x$

$\pi(v) = v \ \forall v \in S$ $\implies$ $\pi \cdot x = x$

$(X, \cdot)$ a Nominal Set

"$\cdot$" a $\mathcal{G}_f(\mathcal{V})$-action & every $x \in X$ finitely supported

$x, y$ in the same orbit of $(X, \cdot)$

if there is $\sigma$ with $\sigma \cdot x = y$.

$\mathcal{V}^2 + 1 \cong$

$\begin{align*}
(b, c) & \quad \quad \quad \quad c \\
\quad \quad \quad \quad b \\
(a, b) & \quad \quad \quad \quad a \\
\quad \quad \quad \quad (a, c)
\end{align*}$

Either infinite

or singleton

Thorsten Wißmann

December 1, 2015
What is it good for?

Instances of regular behaviours with names:

- Regular λ-trees
  \[ L_X = \mathcal{V} + \mathcal{V} \times X + X \times X \]

- Regular λ-trees modulo α-equivalence
  \[ L_\alpha X = \mathcal{V} + [\mathcal{V}]X + X \times X \]

- Regular Nominal Automata
  \[ KX = 2 \times X^\mathcal{V} \times [\mathcal{V}]X \]
Instances of regular behaviours with names:

- Regular $\lambda$-trees
  \[
  LX = \nu + \nu \times X + X \times X
  \]
- Regular $\lambda$-trees modulo $\alpha$-equivalence
  \[
  L_\alpha X = \nu + [\nu]X + X \times X
  \]
- Regular Nominal Automata
  \[
  KX = 2 \times X^\nu \times [\nu]X
  \]
Instances of regular behaviours with names:

- Regular $\lambda$-trees
  \[ L_X = \nu + \nu \times X + X \times X \]

- Regular $\lambda$-trees modulo $\alpha$-equivalence
  \[ L_\alpha X = \nu + [\nu]X + X \times X \]

- Regular Nominal Automata
  \[ K_X = 2 \times X^\nu \times [\nu]X \]

How to prove them being rational fixpoints of appropriate endofunctors on nominal sets?
What is it good for?

Instances of regular behaviours with names:
- Regular λ-trees
  \[ \text{Lifting of a Set-functor (Part 1)} \]
  \[
  LX = \nu + \nu \times X + X \times X
  \]
- Regular λ-trees modulo α-equivalence
  \[
  L_\alpha X = \nu + [\nu]X + X \times X
  \]
- Regular Nominal Automata
  \[ \text{Quotient of a lifting (Part 2)} \]
  \[
  KX = 2 \times X^\nu \times [\nu]X
  \]

How to prove them being rational fixpoints of appropriate endofunctors on nominal sets?
Part 1: Localizable Liftings
Regular Behaviours in Set

\[ F : \text{Set} \rightarrow \text{Set} \]

- finite
- locally finite
- \( \varrho F \)
- arbitrary
- \( \nu F \)

Conditions?

Group action?

Adámek, Milius, Velebil’06; Milius’10
Regular Behaviours in Set ... and in Nom

\[ F : \text{Set} \rightarrow \text{Set} \]

- finite
- locally finite
- arbitrary

\[ \varrho F, \nu F \]

\[ F : \text{Nom} \rightarrow \text{Nom} \]

- orbit-finite
- locally orbit-finite
- arbitrary

\[ \varrho \bar{F}, \nu \bar{F} \]

Adámek, Milius, Velebil’06; Milius’10
Regular Behaviours in Set \dots and in Nom

If $F : \text{Set} \to \text{Set}$ \dots lifts to \dots $\bar{F} : \text{Nom} \to \text{Nom}$

\begin{align*}
\text{finite} & \neq \text{orbit-finite} \\
\text{locally finite} & \neq \text{locally orbit-finite} \\
\varrho F & \neq \varrho \bar{F} \\
\nu F & \neq \nu \bar{F}
\end{align*}

Conditions? Group action?

Adámek, Milius, Velebil’06; Milius’10
Liftings

$G_f(\mathcal{V})$-action on $X$

$T$-algebra structure on $X$ for the monad $T = G_f(\mathcal{V}) \times _{-}$

Liftings $\iff$ Distributive Laws

\[
\begin{array}{ccc}
\text{Set}^T & \xrightarrow{\bar{F}} & \text{Set}^T \\
U \downarrow & & \downarrow U \\
\text{Set} & \xrightarrow{F} & \text{Set}
\end{array}
\]

$\lambda : TF \to FT$

preserving monad structure
Properties of liftings of $\mathcal{S}_f(V) \times \_ \text{ over } F : \text{Set}$ (1)

$\bar{F}$ Nom-restricting

$\bar{F}$ maps nominal sets to nominal sets.

Examples

- Closed under finite products, coproducts, composition.
- For $(Y, \cdot)$ non-nominal, $KX = Y$ not Nom-restricting.
Properties of liftings of $\mathcal{S}_f(\mathcal{V}) \times _\_ \text{ over } F : \text{Set}$ \(\lambda : \mathcal{S}_f(\mathcal{V}) \times F_\_ \rightarrow F(\mathcal{S}_f(\mathcal{V}) \times _)\) localizable

For each $W \subseteq \mathcal{V}$, $\lambda$ restricts to $\lambda : \mathcal{S}_f(W) \times F_\_ \rightarrow F(\mathcal{S}_f(W) \times _)$

Examples

- Closed under finite products, coproducts, composition, constants.
- For $F = \text{Id}_{\text{Set}}$, $\lambda(\pi, x) = (g \cdot \pi \cdot g^{-1})$ not localizable.
Assumptions

Assumption: $\tilde{F} : \text{Nom} \rightarrow a$ localizable lifting, i.e.

1. $\tilde{F}$ comes from a Nom-restricting distributive law $\lambda$ over $F = U\tilde{F}D$.
2. This $\lambda$ is localizable
Assumptions

Assumption: \( \bar{F} : \text{Nom} \rightarrow \text{a localizable lifting, i.e.} \)

1. \( \bar{F} \) comes from a Nom-restricting distributive law \( \lambda \) over \( F = U \bar{F} D. \)
2. This \( \lambda \) is localizable

Examples

- Constants, Identity.
- Closed under finite products, coproducts, composition.
- In particular: Polynomials in Nom
  \( LX = V + V \times X + X \times X \)
- For the strength of any finitary \( F : \text{Set} \) canonically defines a localizable lifting to Nom
**Lemma**

If for $c : C \to \bar{F}C$, the underlying $c : C \to FC$ is lfp in Set, then $c : C \to \bar{F}C$ is lfp in Nom.
LFP in Set vs LFP in Nom

**Lemma**

If for $c : C \to \tilde{F}C$, the underlying $c : C \to FC$ is lfp in Set, then $c : C \to \tilde{F}C$ is lfp in Nom.

**Lemma**

If $c : C \to \tilde{F}C$, with $C$ orbit-finite, then the underlying $c : C \to FC$ is lfp in Set.
**Lemma**

If for $c : C \rightarrow \bar{F}C$, the underlying $c : C \rightarrow FC$ is lfp in Set, then $c : C \rightarrow \bar{F}C$ is lfp in Nom.

**Lemma**

If $c : C \rightarrow \bar{F}C$, with $C$ orbit-finite, then the underlying $c : C \rightarrow FC$ is lfp in Set.

**Corollary**

$c : C \rightarrow \bar{F}C$ lfp in Nom iff the underlying $c : C \rightarrow FC$ is lfp in Set.
Lemma

$(\varrho F, r)$ carries a canonical group action making $r$ equivariant.
Lemma

\((\varrho F, r)\) carries a canonical group action making \(r\) equivariant.

Proof.

\[
\begin{align*}
\mathcal{G}_f(\mathcal{V}) \times \varrho F & \xrightarrow{\text{id} \times r} \mathcal{G}_f(\mathcal{V}) \times F(\varrho F) \\
& \xrightarrow{\lambda_{\varrho F}} F(\mathcal{G}_f(\mathcal{V}) \times \varrho F)
\end{align*}
\]

is lfp because \(\lambda\) is localizable.

\(\nu F\) has canonical \(\mathcal{G}_f(\mathcal{V})\)-set structure (Bartels’04; Plotkin, Turi’97)

This map is just the restriction to \(\varrho F\).
Coinduction

Definition: Coalgebra iteration

For $c : C \to HC$ put $c^{(n+1)} \equiv (C \xrightarrow{c^{(n)}} H^n C \xrightarrow{H^n c} H^{n+1} C)$.

Lemma

Let $H : 	ext{Set} \to \text{Set}^{\text{Set}}$ be finitary. If for $H$-coalgebras $(C, c)$ and $(D, d)$

\[
\begin{array}{cccccc}
X & \xrightarrow{p_1} & C & \xrightarrow{c^{(n)}} & H^n C \\
& \downarrow{p_2} & \downarrow{c^{(n)}} & \downarrow{H^n !} & \\
D & \xrightarrow{d^{(n)}} & H^n D & \xrightarrow{H^n !} & H^n 1
\end{array}
\]

commutes for all $n < \omega$, then $c^\dagger \cdot p_1 = d^\dagger \cdot p_2$. 

Thorsten Wißmann
December 1, 2015
13 / 28
Lemma

Any $t \in \varrho F$ is supported by

$$s(t) = \bigcup_{n \geq 0} \text{supp}(r^{(n)}(t))$$

where $r^{(n)} : \varrho F \to F^n(\varrho F)$

and where the support of $r^{(n)}(t)$ is taken in $\bar{F}^nD(\varrho F)$. 
Lemma

Any $t \in \varrho F$ is supported by

$$s(t) = \bigcup_{n \geq 0} \text{supp}(r^{(n)}(t))$$

where $r^{(n)} : \varrho F \to F^n(\varrho F)$

and where the support of $r^{(n)}(t)$ is taken in $\bar{F}^nD(\varrho F)$.

Lemma

For any $t \in \varrho F$, $s(t)$ is finite.
Universal Property

Theorem

The lifted \((\varrho F, r)\) is the rational fixpoint of \(\bar{F}\).
Universal Property

Theorem
The lifted \((\varrho F, r)\) is the rational fixpoint of \(\bar{F}\).

Proof.
Consider \(c : C \to \bar{F}C\) with \(C\) orbit-finite.

1. \(c\) is lfp in Set, then \(c^\dagger : (C, c) \to (\varrho F, r)\) in Set
2. Equivariant \(j : (\varrho F, r) \leftrightarrow (\nu F, \tau)\) in \(\mathcal{G}_f(\mathcal{V})\)-sets
3. Equivariant \(j \cdot c^\dagger : (C, c) \to (\nu F, \tau)\) in \(\mathcal{G}_f(\mathcal{V})\)-sets
4. \(c^\dagger : (C, c) \to (\varrho F, r)\) equivariant
Examples

...
Examples

$$\lambda$$-trees

$$LX = \mathcal{V} + \mathcal{V} \times X + X \times X$$

- $$\lbar{\varrho}L$$ in Nom = rational $$\lambda$$-trees (not modulo $$\alpha$$-equivalence)
- $$\nu L$$ in Set = all $$\lambda$$-trees
- $$\nu\lbar{\varrho}L$$ in Nom = $$\lambda$$-trees involving finitely many variables

Kurz, Petrisan, Severi, de Vries’13

Canonical Liftings
$$\bar{F} : \text{Nom} \xrightarrow{\equiv} F : \text{Set}$$

$$\lbar{\varrho}F$$ in Nom = $$\varrho F$$ with discrete nominal structure
Examples

\(\lambda\)-trees \(LX = \mathcal{V} + \mathcal{V} \times X + X \times X\)

- \(\varrho \bar{L}\) in \(\text{Nom}\) = rational \(\lambda\)-trees (not modulo \(\alpha\)-equivalence)
- \(\nu L\) in \(\text{Set}\) = all \(\lambda\)-trees
- \(\nu \bar{L}\) in \(\text{Nom}\) = \(\lambda\)-trees involving finitely many variables

\(\nu \bar{L}\) in \(\text{Nom}\) = \(\lambda\)-trees involving finitely many variables

Kurz, Petrisan, Severi, de Vries’13

Canonical Liftings \(\bar{F} : \text{Nom}\) of \(F : \text{Set}\)

- \(\varrho \bar{F}\) in \(\text{Nom}\) = \(\varrho F\) with discrete nominal structure

Unordered Trees: \(FX = \mathcal{B}(X) + \mathcal{V}\)

- \(\nu F\) = unordered trees with some leaves labelled in \(\mathcal{V}\)
- \(\varrho F\) = those with finitely many subtrees
- \(\varrho \bar{F}\) = those with renaming of the leaves
Part 2: Quotients of Nom-functors
Regular Behaviours in Nom

\[ F : \text{Nom} \to \text{Nom} \]

orbits-finite

locally orbit-finite

arbitrary

\[ \varrho F \]

\[ \nu F \]
Regular Behaviours in Nom

\[ F : \text{Nom} \rightarrow \text{Nom} \]

\[ \rho F \]

\[ \nu F \]

\[ \text{orbit-finite} \]

\[ \text{locally orbit-finite} \]

\[ \text{arbitrary} \]

\[ H : \text{Nom} \rightarrow \text{Nom} \]

\[ \rho H \]

\[ \nu H \]

\[ \text{orbit-finite} \]

\[ \text{locally orbit-finite} \]

\[ \text{arbitrary} \]
Regular Behaviours in Nom \(\rightarrow\) \(\ldots\) and their quotients

\[
F : \text{Nom} \to \text{Nom} \quad \xrightarrow{q} \quad H : \text{Nom} \to \text{Nom}
\]

\(\varrho F\) \quad \text{orbit-finite}

\(\nu F\) \quad \text{arbitrary}

\(\varrho H\) \quad \text{orbit-finite}

\(\nu H\) \quad \text{arbitrary}

Conditions? Group action?

\(\neq\)
Quotients of coalgebras

**Definition: Quotient**
A quotient from $a : A \to FA$ to $c : C \to HC$:

Some $h : A \to C$ with

![Diagram](image_url)

Theorem
If every orbit-finite $H$-coalgebra is a quotient of an orbit-finite $F$-coalgebra, then $\varrho_H$ is a quotient of $\varrho_F$. How to prove that?
Quotients of coalgebras

\[
F : \text{Nom} \xrightarrow{q} H : \text{Nom}
\]

**Definition: Quotient**

A quotient from \(a : A \to FA\) to \(c : C \to HC\):

\[
\begin{array}{c}
A \xrightarrow{a} FA \xrightarrow{qA} HA \\
\downarrow h & & \downarrow HHh \\
C \xrightarrow{c} HC
\end{array}
\]

some \(h : A \to C\) with

**Theorem**

If every orbit-finite \(H\)-coalgebra is a quotient of an orbit-finite \(F\)-coalgebra, then \(\varrho H\) is a quotient of \(\varrho F\).
Quotients of coalgebras

\[ F : \text{Nom} \xrightarrow{q} H : \text{Nom} \]

**Definition: Quotient**

A quotient from \( a : A \to FA \) to \( c : C \to HC \):

some \( h : A \to C \) with

\[
\begin{array}{c}
A & \xrightarrow{a} & FA & \xrightarrow{qA} & HA \\
h & \downarrow & & & Hh \\
C & \xrightarrow{c} & HC \\
\end{array}
\]

**Theorem**

If every orbit-finite \( H \)-coalgebra is a quotient of an orbit-finite \( F \)-coalgebra, then \( \varrho H \) is a quotient of \( \varrho F \).

**Proof.**

Epi-laws for jointly-epic the families.
Quotients of coalgebras

\[ F : \text{Nom} \xrightarrow{q} H : \text{Nom} \]

**Definition: Quotient**

A quotient from \( a : A \to FA \) to \( c : C \to HC \):

some \( h : A \to C \) with

\[
\begin{array}{ccl}
A & \xrightarrow{a} & FA \\
\downarrow h & & \downarrow qA \\
C & \xrightarrow{c} & HC
\end{array}
\]

**Theorem**

If every orbit-finite \( H \)-coalgebra is a quotient of an orbit-finite \( F \)-coalgebra, then \( \varrho H \) is a quotient of \( \varrho F \).

**Proof.**

Epi-laws for jointly-epic the families.
Constructing a quotient backwards

**Definition**

\[ X < Y = \{(x, y) \in X \times Y \mid \text{supp}(x) \subseteq \text{supp}(y)\} \]

**Substrength of a functor** \( F: s_{X,Y} : FX < Y \to F(X < Y) \),

with \( F \text{outl} \cdot s_{X,Y} = \text{outl} \) (not necessarily natural).
Constructing a quotient backwards

**Definition**

\[ X < Y = \{(x, y) \in X \times Y \mid \text{supp}(x) \subseteq \text{supp}(y)\} \]

**Substrength of a functor** \( F : s_{X,Y} : FX < Y \to F(X < Y) \),

with \( F \text{outl} \cdot s_{X,Y} = \text{outl} \) (not necessarily natural).

**Construction for** \( c : C \to HC \)

\[ B = \max_{x \in C} |\text{supp}(x)| + \max_{x \in C} \min_{y \in FC} |\text{supp}(y)|. \]

\( W \subseteq \mathcal{V}^B \) of tuples with distinct components.

\( F \)-Coalgebra on \( C < W \).
Something like “projective objects” in Nom

Definition: strongly supported

Some $x \in X$ is strongly supported iff

$$\pi \cdot x = x \implies \forall v \in \text{supp}(x) : \pi(v) = v$$

Examples

$W$ is strongly supported. $P_f(V)$ not.

Proposition (Mentioned already in Kurz, Petrisan, Velebil’10)

$X, Y$ nominal sets, $X$ strongly supported, $O \subseteq X$ a choice of one element from each orbit. Then any map $f_0 : O \rightarrow Y$ with

$$\text{supp}(f_0(x)) \subseteq \text{supp}(x)$$

extends uniquely to an equivariant $f : X \rightarrow Y$. 
**Lemma**

There is an equivariant map \( f : C < W \rightarrow FC \) such that:

\[
\begin{array}{ccc}
C < W & \overset{f}{\longrightarrow} & FC \\
\downarrow \text{outl} & & \downarrow q_C \\
C & \overset{c}{\longrightarrow} & HC
\end{array}
\]
There is an equivariant map \( f : C < W \rightarrow FC \) such that:

\[
\begin{array}{ccc}
C < W & \xrightarrow{f} & FC \\
\text{outl} & \downarrow & \downarrow q_c \\
C & \xrightarrow{c} & HC
\end{array}
\]

Proposition

\( c : C \rightarrow HC \) is via outl a quotient of the orbit-finite

\[
C < W \xrightarrow{\bar{f}} FC < W \xrightarrow{sc,w} F(C < W)
\]

(where \( \bar{f}(x, w) = (f(x), w) \)).
Applied to our $C < W$

**Lemma**

There is an equivariant map $f : C < W \rightarrow FC$ such that:

![Diagram](image)

**Proposition**

$c : C \rightarrow HC$ is via $\text{outl}$ a quotient of the orbit-finite

\[ C < W \xrightarrow{\bar{f}} FC < W \xrightarrow{s_{C,W}} F(C < W) \]

(where $\bar{f}(x, w) = (f(x), w)$).

**Corollary**

If a finitary $F : \text{Nom}$ has a substrength, and $q : F \rightarrow H$, then $\varrho F \rightarrow \varrho H$ (applying $q$ level-wise).
The only restricting requirement: $F$ having a sub-strength $H$ and $q$: arbitrary

Lemma

1. Identity and constant functors have a sub-strength.
2. The class of functors with a sub-strength is closed under finite products, arbitrary coproducts, and functor composition.
Example: $\lambda$-trees modulo $\alpha$-equivalence

$$LX = \mathcal{V} + \mathcal{V} \times X + X \times X \quad \overset{q}{\rightarrow} \quad L_\alpha X = \mathcal{V} + [\mathcal{V}]X + X \times X$$

Definition: Rational $\alpha$-equivalence class of $\lambda$-trees

$= \text{contains some rational } \lambda \text{-tree}$

$\lambda$-trees

- $\varrho L = \text{rational } \lambda \text{-trees}$
- $\nu L = \lambda \text{-trees with finitely many variables involved}$

$\lambda$-trees modulo $\alpha$-equivalence

- $\varrho L_\alpha = \text{rational } \lambda \text{-trees modulo } \alpha \text{-equivalence}$
- $\nu L_\alpha = \lambda \text{-trees with finitely many } \textbf{free} \text{ variables but possibly } \textbf{infinitely} \text{ many bound variables}$

Kurz, Petrisan, Severi, de Vries'13
Example: Exponentiation

\[ FX = \mathcal{V} \times X \times \coprod_{n \in \mathbb{N}} (\mathcal{V} \times X)^n \xrightarrow{q} \ (\_)^\mathcal{V} \]

**Definition**

\[ \bar{q}_X(a, d, (v_1, x_1), \ldots, (v_n, x_n), b) = \begin{cases} x_i & \text{if } i = \min_{1 \leq j \leq n} (v_j = b) \text{ exists} \\ (a \ b) \cdot d & \text{otherwise.} \end{cases} \]

**Theorem: \( q \) component-wise surjective**

For some \( f \in X^\mathcal{V}, \{a_1, \ldots, a_n\} = \text{supp}(f) \) and \( a \in \mathcal{V} \setminus \text{supp}(f) \), we have

\[ \bar{q}_X(a, f(a), (a_1, f(a_1)), \ldots, (a_n, f(a_n)), b) = f(b) \text{ for all } b \in \mathcal{V}. \]
Example: Automata

Various Kinds of Nominal Automata

- $FX = 2 \times X^\nu$
- $KX = 2 \times X^\nu \times [\mathcal{V}]X$
- $NX = 2 \times \mathcal{P}_f(X^\nu) \times \mathcal{P}_f([\mathcal{V}]X)$
Main Results

Final Coalgebra:
Infinite Behaviours

Nominal Sets:
\( S_f(V) \)-action

Rational Fixpoint:
orbit-finite Behaviours

Thorsten Wißmann  December 1, 2015
Main Results

- Final Coalgebra: Infinite Behaviours
- Nominal Sets: finitely supported $\mathcal{G}_f(\mathcal{V})$-action
- Rational Fixpoint: orbit-finite Behaviours

- If $\bar{F} : \text{Nom} \xrightarrow{\varrho} \text{Set}$ is localizable lifting of $F : \text{Set}$ then $\varrho \bar{F}$ is $\varrho F$ with canonical $\mathcal{G}_f(\mathcal{V})$-action
- If $G : \text{Nom} \xrightarrow{\varrho} \text{Set} \rightarrow \text{Nom}$ is a quotient $H : \text{Nom} \xrightarrow{\varrho} \text{Set}$ with a substrength then $\varrho G$ is a quotient of $\varrho H$
Open Questions

About Localizable Liftings
- Is every non-localizable Lifting isomorphic to localizable one?
- If not, are there applications of non-localizable liftings?

About Substrengths
- Rational Fixpoint of quotients of functors without substrength?
- Are there applications?


