

# Recursion Schemes

**folds** (tear down a structure)

$$\text{algebra } f a \rightarrow \text{Fix } f \rightarrow a$$

**unfolds** (build up a structure)

$$\text{coalgebra } f a \rightarrow a \rightarrow \text{Fix } f$$



<b>generalized</b> $(f w \rightarrow w f) \rightarrow (f (w a) \rightarrow \beta)$	<b>catamorphism</b> $f a \rightarrow a$	<b>anamorphism</b> $a \rightarrow f a$	<b>generalized</b> $(m f \rightarrow f m) \rightarrow (a \rightarrow f (m \beta))$
	<b>prepromorphism*</b> ... after applying a NatTrans $(f a \rightarrow a) \rightarrow (f \rightarrow f)$	<b>postpromorphism*</b> ... before applying a NatTrans $(a \rightarrow f a) \rightarrow (f \rightarrow f)$	
	<b>paramorphism*</b> ... with primitive recursion $f (\text{Fix } f \times a) \rightarrow a$	<b>apomorphism*</b> ... returning a branch or single level $a \rightarrow f (\text{Fix } f \vee a)$	
	<b>zygomorphism*</b> ... with a helper function $(f b \rightarrow b) \rightarrow (f (b \times a) \rightarrow a)$	<b>g apomorphism</b> $(b \rightarrow f b) \rightarrow (a \rightarrow f (b \vee a))$	
<b>g histomorphism</b> $(f h \rightarrow h f) \rightarrow (f (w a) \rightarrow a)$	<b>histomorphism</b> ... with prev. answers it has given $f (\text{Cofree } f a) \rightarrow a$	<b>futumorphism</b> ... multiple levels at a time $a \rightarrow f (\text{Free } f a)$	<b>g futumorphism</b> $(h f \rightarrow f h) \rightarrow (a \rightarrow f (m a))$

**refolds** (build up then tear down a structure)

$$\text{algebra } g b \rightarrow (f \rightarrow g) \rightarrow \text{coalgebra } f a \rightarrow a \rightarrow b$$

**hylomorphism**  
cata; ana

<b>dynamorphism</b> histo; ana	<b>codynamorphism</b> cata; futu	<b>generalized</b> apply the generalizations for both the relevant fold and unfold
<b>chronomorphism</b> histo; futu		
<b>Elgot algebra</b> ... may short-circuit while building cata; $a \rightarrow b \vee f a$	<b>coElgot algebra</b> ... may short-circuit while tearing $a \times g b \rightarrow b$ ; ana	

**reunfolds** (tear down then build up a structure)  
 $\text{coalgebra } g b \rightarrow (a \rightarrow b) \rightarrow \text{algebra } f a \rightarrow \text{Fix } f \rightarrow \text{Fix } g$

<b>metamorphism</b> ana; cata	<b>generalized</b> apply ... both ... [un]fold
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**others**

**synchronomorphism**

???

**exomorphism**

???

**mutumorphism**

... can refer to each other's results

$$(f (a \times b) \rightarrow a) \rightarrow (f (a \times b) \rightarrow b)$$

Stolen from Edward Kmett's <http://comonad.com/reader/2009/recursion-schemes/>

\* This gives rise to a family of related recursion schemes, modeled in recursion-schemes with distributive law combinators

These can be combined in various ways. For example, a “zygohistomorphic prepromorphism” combines the zygo, histo, and prepro aspects into a signature like  $(f b \rightarrow b) \rightarrow (f \rightarrow f) \rightarrow (f (w (b \times a)) \rightarrow a) \rightarrow \text{Fix } f \rightarrow a$