Lösungsversuch Probeklausur
ThProg SS25

Alles ohne Gewähr!

Rückfragen und gefundene Fehler an

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A1 1.

(a) Nein, weil $P_{f(g(x))} = x^6 \not> x^6 = P_{g(f(x))}$

(b) Ja, weil $P_{f(g(x))} = 2\cdot(2x+1) = 4x+2$

und $P_{g(f(x))} = 2(2x)+1 = 4x+1$

$$4x+2 > 4x+1$$

für jede Domäne.

2.

(a)

$$\begin{tikzcd}
 & f(g(x \multimap y)) \arrow[dl, "(3)"'] \arrow[dr, "(1)"] & \\
f(g(y) \multimap g(x)) \arrow[d, "(2)"] & & g(f(x \multimap y)) \arrow[d] \\
f(g(y)) \multimap f(g(x)) \arrow[d, "(1)"] & & g(f(x) \multimap f(y)) \arrow[ddl, "(3)"] \\
g(f(y)) \multimap f(g(x)) \arrow[d, "(1)"] & & \\
g(f(y)) \multimap g(f(x) & &
\end{tikzcd}$$

$$\begin{tikzcd}
 & x' \multimap (x \multimap (y \multimap z)) \arrow[dl, "(4)"'] \arrow[dr, "(4)"] & \\
x' \multimap ((x \multimap y) \multimap z) \arrow[d, "(4)"] & & (x' \multimap x) \multimap (y \multimap z) \arrow[d, "(4)"] \\
(x' \multimap (x \multimap y)) \multimap z \arrow[rr, "(4)"] & & ((x' \multimap x) \multimap y) \multimap z
\end{tikzcd}$$

(b) Recall: Kritisches Paar

Betrachte zwei Regeln $l_1 \to_o r_1$, $l_2 \to_o r_2$

ggf mit Umbenennung sicherstellen, dass $FV(l_1) \cap FV(l_2) = \emptyset$!

Sei $l_1 = C(t)$ mit t nicht Variable.

Wenn $\sigma = mgu(t, l_2)$ existiert ist ein KP:

$$\begin{tikzcd}
 & l_1\sigma \arrow[dl] \arrow[dr] & \\
r_1\sigma & & C(r_2)\sigma
\end{tikzcd}$$

Betrachte (2) und (4)

$$f(x' \multimap y') \to_o f(x') \multimap f(y') \qquad (\hat{=}\ l_1 \to_o r_1)$$

$$x \multimap (y \multimap z) \to_o (x \multimap y) \multimap z \qquad (\hat{=}\ l_2 \to_o r_2)$$

$$t = x' \multimap y', \quad C(\cdot) = f(\cdot)$$

$$\sigma = mgu(x' \multimap y', x \multimap (y \multimap z)) = [x' \mapsto x, y' \mapsto y \multimap z]$$

$\Rightarrow$ KP ist:

$$\begin{tikzcd}
 & f(x \multimap (y \multimap z)) \arrow[dl, "(2)"'] \arrow[dr, "(4)"] & \\
f(x) \multimap f(y \multimap z) \arrow[d, "(2)"] & & f((x \multimap y) \multimap z) \arrow[d, "(2)"] \\
f(x) \multimap (f(y) \multimap f(z)) & & f(x \multimap y) \multimap f(z)
\end{tikzcd}$$

$$(f(x) \multimap f(y)) \multimap f(z)$$

Betrachte (3) und (4)

$t = x' \multimap y' \qquad C(\cdot) = g(\cdot)$

$\sigma = \mathrm{mgu}\,(x' \multimap y',\ x \multimap (y \multimap z)) = [\, x' \mapsto x,\ y' \mapsto y \multimap z \,]$

$\Rightarrow$ KP ist

$$\begin{tikzcd}[column sep=small]
 & g(x \multimap (y \multimap z)) \arrow[dl, "(3)"'] \arrow[dr, "(4)"] & \\
g(x) \multimap g(y \multimap z) \arrow[d, "(3)"] & & g((x \multimap y) \multimap z) \arrow[d, "(3)"] \\
g(x) \multimap (g(y) \multimap g(z)) \arrow[ddr, "(4)"] & & g(x \multimap y) \multimap g(z) \arrow[d, "(3)"] \\
 & & (g(x) \multimap g(y)) \multimap g(z)
\end{tikzcd}$$

3. Das System ist nicht SN, denn es gibt die unendliche Reduktionssequenz:

$$\begin{tikzcd}
x \multimap f(y) \arrow[d, "(5)"] & & f(f(x) \multimap f(y)) \arrow[d, "(2)"] \\
f(x \multimap y) \arrow[r, "(2)"] & f(x) \multimap f(y) \arrow[ur, "(5)"] & f(f(x)) \multimap f(y) \arrow[d, "(5)"] \\
 & & {}
\end{tikzcd}$$

A2

1. Ziel $\vdash \lambda x\, l\, u\, f.\ l\,(f\,x\,u)\,f : \forall a.\ a \to \mathbb{L}_a \to \mathbb{L}_a$

Regeln:

$\forall_i, \to_i, \to_i, \forall_i, \to_i, \to_i$

2.

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash l : \forall r.\ r \to (a \to r \to r) \to r}(Ax)}{\Gamma \vdash l : r \to (a \to r \to r) \to r}(\forall_e) \quad \dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : a \to r \to r}(Ax) \quad \dfrac{}{\Gamma \vdash x : a}(Ax)}{\Gamma \vdash f x : r \to r}(\to_e) \quad \dfrac{}{\Gamma \vdash u : r}(Ax)}{\Gamma \vdash f x u : r}(\to_e)}{\Gamma \vdash l\,(f x u) : (a \to r \to r) \to r}(\to_e) \quad \dfrac{}{\Gamma \vdash f : a \to r \to r}(Ax)$$

$$\overline{\Gamma := x : a,\ l : \mathbb{L}_a,\ u : r,\ f : a \to r \to r \vdash l\,(f x u)\, f : r}\ (\to_e)$$

3.

$rev : \forall a.\ \mathbb{L}_a \to \mathbb{L}_a$

$rev = \lambda l.\ l\ nil\ snoc$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash l : \forall r.\ r \to (a \to r \to r) \to r}(Ax)}{\Gamma' \vdash l : \mathbb{L}_a \to (a \to \mathbb{L}_a \to \mathbb{L}_a) \to \mathbb{L}_a}(\forall_e) \quad \dfrac{\dfrac{}{\Gamma' \vdash nil : \forall a.\ \mathbb{L}_a}(Ax)}{\Gamma' \vdash nil : \mathbb{L}_a}(\forall_e)}{\Gamma' \vdash l\ nil : (a \to \mathbb{L}_a \to \mathbb{L}_a) \to \mathbb{L}_a}(\to_e) \quad \dfrac{\dfrac{}{\Gamma' \vdash snoc : \forall a.\ a \to \mathbb{L}_a \to \mathbb{L}_a}(Ax)}{\Gamma' \vdash snoc : a \to \mathbb{L}_a \to \mathbb{L}_a}(\forall_e)}{\Gamma' := \Gamma, l : \mathbb{L}_a \vdash l\ nil\ snoc : \mathbb{L}_a}(\to_e)$$

$$\dfrac{\dfrac{\Gamma' := \Gamma, l : \mathbb{L}_a \vdash l\ nil\ snoc : \mathbb{L}_a}{\Gamma \vdash \lambda l.\ l\ nil\ snoc : \mathbb{L}_a \to \mathbb{L}_a}(\to_i)}{\Gamma \vdash \lambda l.\ l\ nil\ snoc : \forall a.\ \mathbb{L}_a \to \mathbb{L}_a}(\forall_i)$$

A3

1.

$$\text{swap} : \text{HList}\ a\ b \to \text{HList}\ b\ a$$
$$\text{swap HNil} = \text{HNil}$$
$$\text{swap (ConsA x xs)} = \text{ConsB x (swap xs)}$$
$$\text{swap (ConsB x xs)} = \text{ConsA x (swap xs)}$$

Z: $\forall zs.\ \text{swap(swap zs)} = zs$

Fall zs = HNil:

$$\text{swap(swap HNil)} = \text{swap HNil} = \text{HNil} \checkmark$$

Fall zs = ConsA x xs

IV: swap (swap xs) = xs

$$\text{swap (swap (ConsA x xs))}$$
$$= \text{swap (Cons B (swap xs))}$$
$$= \text{Cons A x (swap (swap xs))}$$
$$\overset{IV}{=} \text{ConsA x xs} \checkmark$$

Fall zs = ConsB x xs

IV: swap(swap xs) = xs

$$\text{swap (swap (ConsB x xs))}$$
$$= \text{swap (ConsA (swap xs))}$$
$$= \text{Cons B x (swap (swap xs))}$$
$$\overset{IV}{=} \text{Cons B x xs} \checkmark$$

$\square$

2. $Z: \forall f, g, zs.\ \text{mapA } f\ (\text{mapB } g\ zs) = \text{mapB } g\ (\text{mapA } f\ zs)$

Induktion über zs:

Fall $zs = \text{HNil}$:

$$\text{mapA } f\ (\text{mapB } g\ \text{HNil}) = \text{mapA } f\ \text{HNil} = \text{HNil}$$
$$\text{mapB } g\ (\text{mapA } f\ \text{HNil}) = \text{mapB } g\ \text{HNil} = \text{HNil} \quad \checkmark$$

Fall $zs = \text{ConsA } x\ xs$

$$\begin{aligned}
&\text{mapA } f\ (\text{mapB } g\ (\text{ConsA } x\ xs))\\
&= \text{mapA } f\ (\text{ConsA } x\ (\text{mapB } g\ xs))\\
&= \text{ConsA } (f\ x)\ (\text{mapA } f\ (\text{mapB } g\ xs))\\
&\overset{IV}{=} \text{ConsA } (f\ x)\ (\text{mapB } g\ (\text{mapA } f\ xs))\\
&= \text{mapB } g\ (\text{ConsA } (f\ x)\ (\text{mapA } f\ xs))\\
&= \text{mapB } g\ (\text{mapA } f\ (\text{ConsA } x\ xs)) \quad \checkmark
\end{aligned}$$

$\square$

3. $\text{foldH} :: r \to (a \to r \to r) \to (b \to r \to r) \to \text{HList } a\ b \to r$

$$\begin{aligned}
&\text{foldH } n\ f\ g\ \text{HNil} = n\\
&\text{foldH } n\ f\ g\ (\text{ConsA } x\ xs) = f\ x\ (\text{foldH } n\ f\ g\ xs)\\
&\text{foldH } n\ f\ g\ (\text{ConsB } x\ xs) = g\ x\ (\text{foldH } n\ f\ g\ xs)
\end{aligned}$$

4. $\text{mapA} :: (a \to c) \to \text{HList } a\ b \to \text{HList } c\ b$

$$\begin{aligned}
\text{mapA } f = \text{foldH HNil } &(\lambda x\ xs.\ \text{ConsA } (f\ x)\ xs)\\
&(\lambda x\ xs.\ \text{ConsB } x\ xs)
\end{aligned}$$

$mapA : (b \to c) \to HList\ a\ b \to HList\ a\ c$

$$mapA\ g = foldH\ HNil\ (\lambda x\ xs.\ ConsA\ x\ xs)$$
$$(\lambda x\ xs.\ ConsB\ (g\ x)\ xs)$$

$\boxed{A4}$ 2-dimensionale streams:

$$s = \begin{cases} [a_1, a_2, a_3, \ldots] \\ [b_1, b_2, b_3, \ldots] \\ [c_1, c_2, c_3, \ldots] \\ \vdots \end{cases} \qquad transpose\ s = \begin{cases} [a_1, b_1, c_1, \ldots] \\ [a_2, b_2, c_2, \ldots] \\ [a_3, b_3, c_3, \ldots] \\ \vdots \end{cases}$$

1. $map\ hd\ (transpose\ s) = hd\ s$

   $map\ tl\ (transpose\ s) = transpose\ (tl\ s)$

2. Z: $\forall s.\ transpose\ (transpose\ s) = s$

Wir zeigen: $R \subseteq Stream\ (Stream\ a) \times Stream\ (Stream\ a)$

$R = \{(transpose(transpose\ s), s) \mid s : Stream\ (Stream\ a)\}$

ist Bisimulation:

1. hd

$$hd(transpose\ (transpose\ s)) = map\ hd\ (transpose\ s)$$
$$\overset{(1.)}{=} hd\ s.$$

2. tl

$tl\ (transpose\ (transpose\ s))$

$= \text{transpose}\ (\text{map}\ tl\ (\text{transpose}\ s))$

$= \text{transpose}\ (\text{transpose}\ (tl\ s))$

$R\ tl\ s.$ $\square$

3. Für z.B.

$$s = \begin{cases} [a_1, a_2, a_3, \ldots] \\ [b_1, b_2, b_3, \ldots] \\ [c_1, c_2, c_3, \ldots] \\ \vdots \end{cases}$$

gilt $\quad f\ s = [a_1, a_2, b_1, a_3, c_1, a_4 \ldots$

Also: f alterniert zwischen den Elementen der ersten Zeile u. ersten Spalte.

A5 q_3, q_4 unerreichbar!

q_0		0	0	0	0	0	0
q_1		0	0	0	0	0	0
q_2					1	1	1
q_5					1	1	1
q_6					1	1	1
q_7						2	2
q_8							2
q_9							

$\Rightarrow$ merge q_0, q_1 und q_2, q_5, q_6

$\Rightarrow$